

# Gluino Contribution to Radiative $B$ Decays: New Operators, Organization of QCD Corrections and Leading Order Results ‡

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**Abstract.** The gluino-induced contribution to the decay  $b \rightarrow s\gamma$  is investigated in supersymmetric frameworks with generic sources of flavour violation. It is emphasized that the operator basis of the standard model effective Hamiltonian is enlarged and that a suitable redefinition allows to realize the usual scheme of leading and next-to-leading logarithmic contributions when QCD corrections are included. The effects of the leading order QCD corrections on the inclusive branching ratio for  $b \rightarrow s\gamma$  are shown. Constraints on supersymmetric sources of flavour violation are derived.

## 1. Introduction

Like the usual weak interactions, also new physics interactions give rise to an ‘effective’ Hamiltonian operator at low energies (well below  $M_W$ ). It only contains the ‘active’ particles, that is quarks, leptons, photons and gluons. The details of the heavy (new) particles and forces on them acting disappear and their properties manifest themselves in the form and the strength of the various operators in the effective Hamiltonian.

In the standard model (SM), the only operator at tree level comes from  $W$ -exchange:

$$\mathcal{H}_{eff} \sim G_F (\bar{q}_L \gamma_\mu q'_L) (\bar{q}_L'' \gamma^\mu q_L''') \quad (1)$$

where the  $q$  are the various quark flavours and factors like the CKM matrix elements have been dropped. The form and strength of the  $W$  couplings are embodied in  $G_F$  and the left-handedness of the quarks. Because of the various gluonic corrections, the number of active operators is much larger than just the above four-Fermi term. Dimensional arguments and renormalization group techniques allow to classify them systematically.

If there are new forces, they will yield not only different couplings strengths, but also different forms for the interactions. This implies that there are in general more operators than the one in eq. (1) and also more than those obtained in the SM after QCD corrections have been included.

‡ Presented by D. Wyler at “Heavy Flavour 2000” (Durham); based on Ref. [1]

In supersymmetric models, there are many sources of flavour violations relevant to the decay  $b \rightarrow s\gamma$  [2]. We shall consider here only those due to a flavour violating gluino-quark-squark vertex [3], whose impact on the decay  $b \rightarrow s\gamma$  was first analyzed in ref. [4]. In general, the gluino contribution to this decay is the largest, unless it is assumed that the only source of flavour violation is encoded in the superpotential and that the deriving electroweak-scale flavour-violating parameters in the scalar sector are small. In this case, the gluino contribution can be neglected, unless  $\tan\beta$  is large [5]. A complete calculation of the  $b \rightarrow s\gamma$  rate, including corrections up to the next-to-leading order (NLO) in QCD, exists for a specific class of models in which the gluino contribution can be neglected [6]. Until recently, no study existed on the interplay between the gluino contribution to the decay  $b \rightarrow s\gamma$  and QCD corrections, not even at the leading order (LO), in spite of the potentially important role that the flavour-violating vertices gluino-quark-squarks may have for model building (see for example Ref. [7]).

The gluino contribution to the decay  $b \rightarrow s\gamma$  exhibits two special features, when QCD corrections are implemented. First, gluinos couple to both left- and right-handed quarks. Thus, many more operators than in the SM, in which only left-handed quarks are coupled to the  $W$  boson, are expected. Secondly, the presence of the strong coupling  $\alpha_s$  in the gluino contribution and in the QCD corrections immediately raises the question of how to order the different powers of  $\alpha_s$  and whether to include them in the gluino-induced operators or in their Wilson coefficients. These two difficulties necessitate a dedicated study of the QCD corrections to this contribution.

## 2. Ordering the QCD perturbation expansion and the effective Hamiltonian

In the SM, rare  $B$  decays are induced by loops with  $W$  bosons; the coupling strength is always  $G_F$ . The corrections are due to gluons and other light particles and give rise to powers of the large logarithmic factor  $L = \log(m_b^2/M_W^2)$ . In the decay  $b \rightarrow s\gamma$  only loops with gluons contribute, and thus powers of  $L$  and  $\alpha_s$  are related as follows [8]:

$$\begin{aligned} \text{LO:} \quad & G_F (\alpha_s L)^N, \quad (N = 0, 1, \dots) \\ \text{NLO:} \quad & G_F \alpha_s (\alpha_s L)^N. \end{aligned}$$

These terms are summed (over  $N$ ) using renormalization group techniques. This results in an effective Hamiltonian

$$\mathcal{H}_{eff}^W = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i(\mu) \mathcal{O}_i(\mu), \quad (2)$$

where  $V_{tb}$  and  $V_{ts}$  are elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix and  $\mu$  is the ‘scale’. The Wilson coefficients  $C_i$  contain all dependence on the heavy degrees of freedom, whereas the operators  $\mathcal{O}_i$  depend on light fields only. When all order QCD corrections are included, the Hamiltonian is independent of the scale  $\mu$ . At a specific order in QCD, however, there is a residual dependence on  $\mu$ . In practice, this scale must be chosen to be around  $m_b$ .

The operators relevant to radiative  $B$  decays can be divided into two classes: four-fermion current-current operators and magnetic operators

$$\mathcal{O}_M = \frac{g}{16\pi^2} \bar{m}_b(\mu) (\bar{s}\sigma^{\mu\nu} P_R b) V_{\mu\nu}, \quad (3)$$

where  $V_{\mu\nu}$  is the field strength of a photon or gluon. All these operators are of dimension six.

It is by now well known that, in the SM, a consistent calculation for  $b \rightarrow s\gamma$  at LO (or NLO) precision requires three steps:

- 1) a matching calculation of the full SM theory with the effective theory at the scale  $\mu = \mu_W$  to order  $\alpha_s^0$  (or  $\alpha_s^1$ ) for the Wilson coefficients, where  $\mu_W$  denotes a scale of order  $M_W$  or  $m_t$ ;
- 2) a renormalization group treatment of the Wilson coefficients using the anomalous-dimension matrix to order  $\alpha_s^1$  (or  $\alpha_s^2$ );
- 3) a calculation of the operator matrix elements at the scale  $\mu = \mu_b$  to order  $\alpha_s^0$  (or  $\alpha_s^1$ ), where  $\mu_b$  denotes a scale of order  $m_b$ .

Matters can be different in other  $B$  decays or when other contributions to  $b \rightarrow s\gamma$  are considered. An example is the decay  $b \rightarrow s\ell\bar{\ell}$ . The first large logarithm  $L = \log(m_b^2/M_W^2)$  arises without the exchange of gluons. This possibility has no correspondence in the  $b \rightarrow s\gamma$  case. Consequently, in the case of  $b \rightarrow s\ell\bar{\ell}$ , the decay amplitude is ordered according to  $G_F L (\alpha_s L)^N$  at the LO in QCD and  $G_F \alpha_s L (\alpha_s L)^N$  at the NLO. To achieve technically the resummation of these terms, it is convenient to redefine certain operators and their Wilson coefficients as follows [9]:

$$\mathcal{O}_i^{new} = \frac{16\pi^2}{g_s^2} \mathcal{O}_i, \quad C_i^{new} = \frac{g_s^2}{16\pi^2} C_i \quad (i = 7, \dots, 10). \quad (4)$$

This redefinition allows us to proceed according to the above three steps when calculating the amplitude of the decay  $b \rightarrow s\ell\bar{\ell}$  [9]. In particular, the one-loop mixing of the operator  $\mathcal{O}_2$  with the operator  $\mathcal{O}_9^{new}$  appears formally at  $\mathcal{O}(\alpha_s)$ .

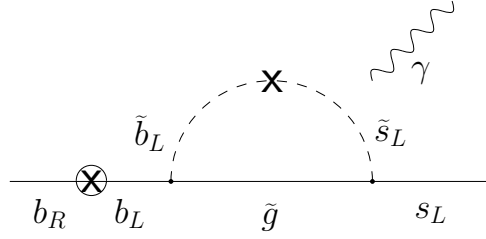
Including gluinos, we can now write the complete effective Hamiltonian as:

$$\mathcal{H}_{eff} = \mathcal{H}_{eff}^W + \mathcal{H}_{eff}^{\tilde{g}}, \quad (5)$$

where  $\mathcal{H}_{eff}^W$  is the SM effective Hamiltonian in (2) and  $\mathcal{H}_{eff}^{\tilde{g}}$  originates after integrating out squarks and gluinos. Note that ‘mixed’ diagrams, which contain, besides a  $W$  boson, also gluinos and squarks, give rise to  $\alpha_s$  corrections to the Wilson coefficients in  $\mathcal{H}_{eff}^W$  (at the matching scale). Such contributions can be omitted in a LO calculation, but they have to be taken into account at the NLO level. As for  $\mathcal{H}_{eff}^{\tilde{g}}$ , the aim is to resum the following terms:

$$\begin{aligned} \text{LO:} \quad & \alpha_s (\alpha_s L)^N, \quad (N = 0, 1, \dots) \\ \text{NLO:} \quad & \alpha_s \alpha_s (\alpha_s L)^N. \end{aligned}$$

While  $\mathcal{H}_{eff}^{\tilde{g}}$  is unambiguous, it is a matter of convention whether the  $\alpha_s$  factors, peculiar of the gluino exchange, should be put into the definition of operators or into the Wilson



**Figure 1.** Diagram mediating the  $b \rightarrow s\gamma$  decay through gluino exchange and contributing to the operator  $\mathcal{O}_{7b,\tilde{g}}$ . A contribution to the primed operator  $\mathcal{O}'_{7b,\tilde{g}}$  is obtained by exchanging  $L \leftrightarrow R$ .

coefficients. It is convenient (and possible) to distribute the factors of  $\alpha_s$  between operators and Wilson coefficients in such a way that the first two of the three steps in the program for the SM calculation also apply to the gluino-induced contribution. This implies one factor of  $\alpha_s^1$  in the definition of the magnetic and chromomagnetic operators and a factor  $\alpha_s^2$  in the definition of the four-quark operators. With this convention, the matching calculation and the evolution down to the low scale  $\mu_b$  of the Wilson coefficients are organized exactly in the same way as in the SM. The anomalous-dimension matrix, indeed, has the canonical expansion in  $\alpha_s$  and starts with a term proportional to  $\alpha_s^1$ . The last of the three steps in the program of the SM calculation requires now an obvious modification: the calculation of the matrix elements has to be performed at order  $\alpha_s$  and  $\alpha_s^2$  at the LO and NLO precision. With this organization of QCD corrections, the SM Hamiltonian  $\mathcal{H}_{eff}^W$  in eq. (2) and the gluino-induced one  $\mathcal{H}_{eff}^{\tilde{g}}$  undergo separate renormalization, which facilitates all considerations.

The list of such redefined operators is lengthy and given in ref. [1]. For illustration only some representative ones are shown here:

- magnetic operators, with chirality violation coming from the  $b$ -quark mass:

$$\mathcal{O}_{7b,\tilde{g}} = e g_s^2(\mu) \overline{m}_b(\mu) (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu}, \quad (6)$$

- magnetic operators in which the chirality-violating parameter is the gluino mass  $m_{\tilde{g}}$ , included in the corresponding Wilson coefficients:

$$\mathcal{O}_{7\tilde{g},\tilde{g}} = e g_s^2(\mu) (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu}, \quad (7)$$

- magnetic operators, with chirality violation signalled by the  $c$ -quark mass:

$$\mathcal{O}_{7c,\tilde{g}} = e g_s^2(\mu) \overline{m}_c(\mu) (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu}, \quad (8)$$

- four-quark operators with vector Lorentz structure:

$$\mathcal{O}_{11,\tilde{g}}^q = g_s^4(\mu) (\bar{s}\gamma_\mu P_L b) (\bar{q}\gamma^\mu P_L q), \quad (9)$$

- four-quark operators with scalar and tensor Lorentz structure:

$$\mathcal{O}_{15,\tilde{g}}^q = g_s^4(\mu) (\bar{s} P_R b) (\bar{q} P_R q). \quad (10)$$

These last four-quark operators are induced by box diagrams only and through the exchange of two gluinos, whereas those with a vector structure are induced by box

and penguin diagrams. For each of these operators there are several similar ones, with different helicity or colour structure. Those with opposite helicity structure, obtained through the exchange  $L \leftrightarrow R$ , are called hereafter “primed” operators.

The four-quark operators in (9) and (10) are formally of higher order in the strong coupling than the magnetic and chromomagnetic operators. The scalar/tensor operators mix at one loop into the magnetic and chromomagnetic operators, giving rise to a large  $\log L$ . Given this fact, the necessity of including  $\mathcal{O}_{7c,\tilde{g}}$  and  $\mathcal{O}_{8c,\tilde{g}}$  in the operator basis becomes clear immediately. Due to these mixing effects, the scalar/tensor operators have to be included in a LO calculation for the decay amplitude. The remaining four-quark operators with vector structure  $\mathcal{O}_{11,\tilde{g}}^q - \mathcal{O}_{14,\tilde{g}}^q$  (and the corresponding primed operators) do not mix at one loop neither into the magnetic and chromomagnetic operators nor into the scalar/tensor four-quark operators. Therefore, these vector four-quark operators become relevant only at the NLO precision.

### 3. Wilson Coefficients at the Decay Scale

At the electroweak scale, the Wilson coefficients of the various operators are obtained by perturbation theory; gluino exchanges such as in fig. 1 contribute. The renormalization program then allows to calculate the coefficients at the relevant scale  $m_b$ . As already mentioned, the two terms  $\mathcal{H}_{eff}^W$  and  $\mathcal{H}_{eff}^{\tilde{g}}$  in the effective Hamiltonian (5) undergo separate renormalization. The anomalous-dimension matrix of the SM operators  $\mathcal{O}_1 - \mathcal{O}_8$  and the evolution of the corresponding Wilson coefficients to the decay scale  $\mu_b$  are very well known and can be found in [10]. The evolution of the gluino-induced Wilson coefficients  $C_{i,\tilde{g}}$  from the matching scale  $\mu_W$  down to the low-energy scale  $\mu_b$  is described by the renormalization group equation:

$$\mu \frac{d}{d\mu} C_{i,\tilde{g}} = C_{j,\tilde{g}}(\mu) \gamma_{ji,\tilde{g}}(\mu). \quad (11)$$

The usual perturbative expansion for the initial conditions of the Wilson coefficients,

$$C_{i,\tilde{g}}(\mu_W) = C_{i,\tilde{g}}^0(\mu_W) + \frac{\alpha_s(\mu_W)}{4\pi} C_{i,\tilde{g}}^1(\mu_W) + \dots, \quad (12)$$

as well as for the elements of  $\gamma_{ji,\tilde{g}}(\mu)$ ,

$$\gamma_{ji,\tilde{g}}(\mu) = \frac{\alpha_s(\mu)}{4\pi} \gamma_{ji,\tilde{g}}^0 + \frac{\alpha_s^2(\mu)}{(4\pi)^2} \gamma_{ji,\tilde{g}}^1 + \dots, \quad (13)$$

is possible because of the choice of including appropriate powers of  $g_s(\mu)$  into the definition of the operators  $\mathcal{O}_{i,\tilde{g}}$ , as discussed previously. The anomalous-dimension matrix  $\gamma_{ji,\tilde{g}}$  is then a  $112 \times 112$  matrix. However, the vectorlike character of QCD and dimensionality arguments reduce it to two identical  $54 \times 54$  matrices. At LO, a further reduction to  $8 \times 8$  matrices occurs. At the low scale, the LO expression for the Wilson coefficients of the dimension-six operators are

$$\begin{aligned} C_{7b,\tilde{g}}(\mu_b) &= \eta^{\frac{39}{23}} C_{7b,\tilde{g}}(\mu_W) + \frac{8}{3} \left( \eta^{\frac{37}{23}} - \eta^{\frac{39}{23}} \right) C_{8b,\tilde{g}}(\mu_W) + R_{7b,\tilde{g}}(\mu_b), \\ C_{8b,\tilde{g}}(\mu_b) &= \eta^{\frac{37}{23}} C_{8b,\tilde{g}}(\mu_W) + R_{8b,\tilde{g}}(\mu_b). \end{aligned} \quad (14)$$

where the remainder functions  $R_{7b,\tilde{g}}(\mu_b)$  and  $R_{8b,\tilde{g}}(\mu_b)$  contain the coefficients of the scalar/tensor operators and are small in this case. At the NLO, also the operators with vector Lorentz structure enter.

#### 4. Flavour violation and results

While the main purpose of this talk was to stress the importance of including and systematically treating the new operators arising in beyond the SM physics, a few specific results should also be mentioned. Gluino exchange gives rise to flavour changes if the quark and squark mixing matrices are different (more precisely: do not commute). This happens when the so-called soft contributions to the squark mass matrix are substantially off-diagonal. Of course the values depend on the specific model; here we introduce phenomenologically the quantities (see ref. [11]):

$$\delta_{LL,ij} = \frac{(m_{d,LL}^2)_{ij}}{m_{\tilde{q}}^2}, \quad \delta_{RR,ij} = \frac{(m_{d,RR}^2)_{ij}}{m_{\tilde{q}}^2}, \quad (i \neq j) \quad (15)$$

which are a measure for flavour violation if  $i \neq j$ . The various  $m^2$  are the elements of the soft mass matrices. The parameters are varied, as usual, in their allowed ranges.

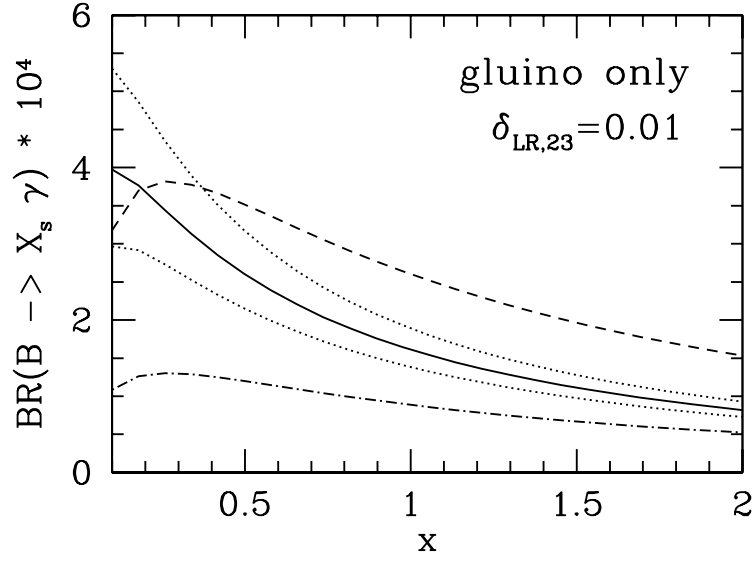
Figs. 2 and 3 exemplify the results of this work. While the first shows how much systematic QCD corrections influence the branching ratio, fig. 3 illustrates its sensitivity to the soft matrix elements.

#### Acknowledgments

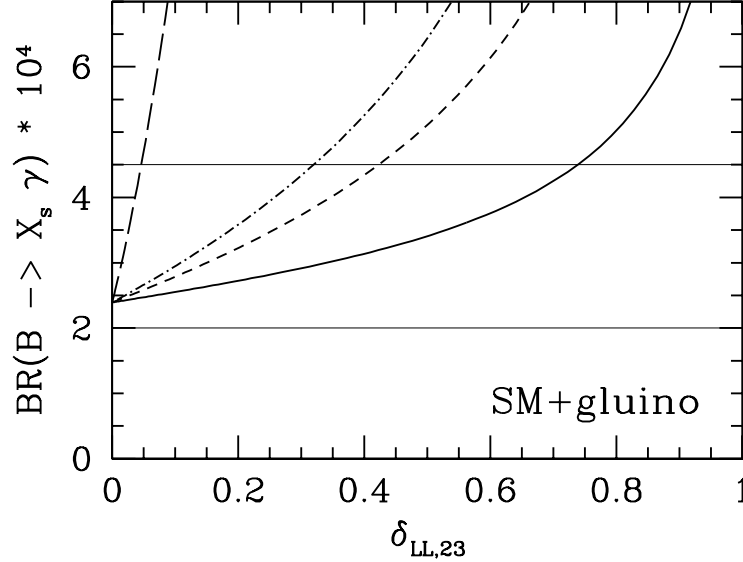
We thank our collaborators Ch. Greub and T. Hurth for a fruitful collaboration. This work was partially supported by the Schweizerischer Nationalfond.

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**Figure 2.** Gluino-induced branching ratio  $\text{BR}(\bar{B} \rightarrow X_s \gamma)$  as a function of  $x = m_g^2/m_q^2$ , obtained when the only source of flavour violation is  $\delta_{LR,23}$  (see text), fixed to the value 0.01, for  $m_{\tilde{q}} = 500$  GeV. The solid line shows the branching ratio at the LO in QCD, for  $\mu_b = 4.8$  GeV and  $\mu_W = M_W$ ; the two dotted lines indicate the range of variation of the branching ratio when  $\mu_b$  spans the interval 2.4–9.6 GeV. Also shown are the values of  $\text{BR}(\bar{B} \rightarrow X_s \gamma)$  when no QCD corrections are included and the explicit factor  $\alpha_s(\mu)$  in the gluino-induced operators is evaluated at 4.8 GeV (dashed line) or at  $M_W$  (dot-dashed line).



**Figure 3.**  $\text{BR}(\bar{B} \rightarrow X_s \gamma)$  vs.  $\delta_{LL,23}$ , when  $\delta_{LL,23}$  and  $\delta_{LR,33}$  are the only sources of chiral-flavour violation. The dependence on  $\delta_{LL,23}$  is shown for different values of  $\delta_{LR,33}$ : 0 (solid line), 0.006 (short-dashed line), 0.01 (dot-dashed line), 0.1 (long-dashed line). The value of  $x = m_g^2/m_q^2$  is fixed to 0.3 and  $m_{\tilde{q}}$  to 500 GeV. The vertical band indicates the experimental constraint.